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Computer Spreadsheet and Investigative Activities: A Case Study of an Innovative Experience

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Abstract: This paper analyses an experience undertaken by a group of teachers, who introduced the computer in a 10th grade mathematics classroom for carrying out problem solving and investigational activities. The main question of interest is the discussion of significant issues and unexpected situations that emerged within this innovative process, specially regarding the reactions of the students. Using a case study methodology [1], the sources of data were interviews with the teachers, visits to the school, and the learning and dissemination materials produced.

Keywords: computers in mathematics education, problem solving, investigations, teachers' beliefs, innovation, new information technologies

The MINERVA Project — Node DEFCUL

The experience described in this study was carried in one of the schools in the MINERVA Project. This national Project, in operation since 1985, was designed for the introduction of New Information Technologies (NIT) in primary, middle and secondary schools, both as topic of interest in itself and related to the teaching of all school subjects. The Node DEFCUL is one of its groups, based at the Department of Education of the College of Sciences of the University of Lisbon. It carries out development, research and extensive teacher training activities. A detailed description of the work of this Pole of the Project was given elsewhere [2] and here we will just review some aspects of its educational approach, based in the principles of active learning and in a concern with broad cultural and educational changes.

The concept of *New Information Technologies* (NIT) implies far reaching ideas than the mere use of electronic hardware. These technologies assume an important role in many social activities, bringing about significant structural changes in ways of working and thinking. They have influenced scientific research, economic planning, goods production, management, communication media, etc. The NIT mean the possibility of automatic information processing and communication. They represent new ways of looking at information and at the process of knowledge development and dissemination, and touch on important values. In practice, most of the work carried out in the schools has developed around the computer (with or without peripherals), but the emphasis is in the general concept of NIT.

One major pedagogical idea about the educational use of computers is that they are essentially regarded as something for the *students to work with.* Of course, teachers need also to be able to deal with computers in many ways, in particular for demonstrations. But the most interesting and innovative aspects of the educational practices that can be expected to develop in schools may have origin on the intensive interactions between students and computers.

The computer is also regarded as a *working tool.* There are other roles that it may fulfil, and they may be of significant educational value. However, it is through its use as a general or subject specific tool, for carrying out investigations and undertaking sophisticated projects, or to perform quite simple tasks, that the computer is expected to find an important place as a cultural artifact influencing human thinking and interpersonal relationships.

In an information-driven society, it becomes necessary to be able to use with ease the computer and other equipment associated with NIT. New educational objectives need to be considered. The youngsters need to develop, since quite early, the ability of knowing where and how to locate information, how to select, interpret, process it and evaluate the results. It is also necessary to reflect about what is its role in social, economical, and cultural processes and what may be misuses and unethical procedures. Quite specially, it is important to be aware of its role in scientific thinking, project design, and more generally, in the development of knowledge.

It should not be surprising, therefore, that *project work*, usually undertaken in groups, has been one of the most important activities within the schools. Students get an opportunity to establish working goals, set strategies and methodologies, collect data and analyse it, and get used to present their results and ideas to large audiences. In mathematics, a common activity is also carrying out investigations, in which students explore the possible relationships between concepts and try to get specific understandings or arrive at broad generalizations (for another example of this kind of activity, see [3]).

The NIT, as a new subject, rise the issue of curricular articulation. It has been felt that instead of setting a new discipline apart to deal with these matters, they should inform deeply in the teaching of all existing school subjects, being *integrated* with them.

These technologies, besides their curricular implications, are also, and more generally, an important factor of *transformation of the school*, yielding the setting of new objectives, new learning situations, new activities, and values. They point towards the reorganization of the spaces, working methods and teacher/student relationships.

The NIT put new demands on the teachers. They need to know how to use these technologies in an effective and confident way. Teachers need to develop new attitudes and abilities, have access to specialised information, receive formal training, and have the possibility of informal interaction with other teachers with similar interests and concerns.

The role of the teacher is changing in a number of aspects. Knowledge is becoming more and more of a dynamic nature, and the active involvement of the learner is critical in the process of its acquisition and development. Teachers have to become experts in supporting this kind of

learning, fostered by the dissemination of information and the availability of sophisticated processing and communication technologies.

The concepts of *active involvement* and *recurrent reflection* in the learning/training process is both relevant to students and teachers. In this Project teachers are not viewed as software developers, but as *curriculum developers* and *project leaders*, both at classroom and school level. This requires the ability to make appropriate use of existing software, both specific and of general purpose. It suggests the need of new approaches to preservice and inservice teacher training, getting them involved in working in their own projects, together in groups at school or local level, and in many instances in cooperation with higher education institutions such as the group that concerns us in this study.

Context of the Experience

The School. This experience took place in a relatively small secondary school located in one of the largest suburbs of Lisbon. Its students are from grades 7 to 11. The number of teachers is about 80. It is a quite recent school, which just opened 7 years ago. It is governed by a Directive Board of five elements who have been in charge since the creation of the school.

The students came from the surroundings and their socioeconomic level is slightly below average, with a great diversity of social backgrounds. The students in this experience were all in the same 10th grade class.

This school has attracted an increasing number of qualified mathematics teachers. At the present time, they constitute a very dynamic group who has taken several initiatives to promote a positive view of the discipline. In general, they have the support of the Directive Board. However, some of the teachers still complain about resistances and lack of pedagogical concerns in some of their colleagues.

The school was involved in the MINERVA Project six years ago. This has been recognized as a decisive factor to rise new pedagogical concerns and attitudes among teachers. It also promoted an important setting to stimulate a bigger participation of students in project work within the school activities and provided stimulus to the up rising of other school projects.

The teachers and their motivations. In the school year of 1989/90, the school had for the first time a group of four mathematics teachers who were doing preservice teaching to complete their graduation and certification requirements. We will refer to them as "practising teachers". Their work at the school was supervised by a senior teacher of the school staff.

The four practising teachers were aged between 24 and 28 years old and had completed four years of academic studies at the University. Three of them were teaching for the first time and only one had some previous experience as a Religion and Moral teacher.

As the supervisor teacher stated, the group had a very enjoyable way of working. They all liked to discuss their points of view and committed themselves to the activities they decided to

carry out. They had very different personal characters and also distinct opinions on education matters, but they respected each others' ideas.

The four practising teachers and the supervisor decided to get involved in a project of introducing calculators in this mathematics class. In a subsequent stage of the work computers were also included. As they reported, the group was, from the first moment, interested in establishing a setting which could help to promote a different sort of mathematical experience to the students. The calculator and then the computer were viewed as good carriers of the innovative strategies that they were looking for.

The work of this group was supported by a training programme on spreadsheets and calculators in mathematics education. Within this programme they had several meetings with practical activities, discussions and exchanges of experiences with teachers of other schools and were visited several times by the project coordinator, often at their request [4].

The practising teachers felt that their lack of teaching experience was a good reason to be gladly open to new ideas and teaching methodologies. They were much concerned with the picture they conceived of the current situation in the teaching and learning of mathematics. They were also very enthusiastic about doing problem solving activities and enhancing students ability to investigate mathematical questions in rich content situations.

They have taken these concerns in their university studies, especially in the previous year, in the mathematics methods course. Other important motivations and inspirations were received through their participation in a mathematics teachers national meeting (PROFMAT). The supervisor always supported their initiatives and was ready to help them to go forward with their projects. She had the caution of making the group reflect upon their actions and understand the possible advantages and risks involved, but she never wanted to dictate their decisions.

The practising teachers emphasized the need for many changes in mathematics education. They believed that the teachers' practices are too often a straight routine consisting in teacher presentation of mathematical concepts, followed by a series of standard exercises where students are supposed to acquire procedural behaviors intended to make them apply the previous information presented. With this kind of exhaustive practice, they found very plausible that students would be deprived of the real opportunity to understand the meaning of mathematical ideas and would possibly perform many calculations without figuring out the reasons for them.

With this in mind and the conviction that plain recepies were not available to create a rich educational environment, the group of teachers embraced some pedagogical guidelines and started out the search for new challenges to themselves and their students.

They wished to give students a less statical view of mathematics and to stress that many mathematical questions demand creative strategies and can be explored in many different ways.

They began with problem solving activities which were supposed to encourage students' work and discussion in groups. This and the opportunities for students to engage in mathematical investigations created the general framework for the introduction of curricular topics.

The first idea of using computers occurred in a discussion among teachers about the possible ways of treating some algebraic issues like equations and inequalities. In the context of their pedagogical concerns, the computer was seen as another ingredient to be added in a natural way. They trusted that it would be a learning facilitator and a powerful tool to shift the emphasis from the calculus to the interpretation of graphs and the translation between different representations of mathematical concepts.

The teachers also alluded to the particular reluctance they felt about the usual way this subject is presented to the students, especially in their text books, because it leads to a heavy and boring use of computations. They thought that the computer could give a good contribution to avoid all that and so they decided to bring it into the classroom.

The students. The students involved in the experience were all in one 10th grade class. This class was under the responsibility of the supervising teacher who shared it with the four practising teachers. Their work was organised in such a way that each of them was alternately the responsible for the class during a sequence of lessons. Nevertheless, for the majority of lessons, the teachers planned the activities together and were frequently all present in the classroom.

The 10th grade class, which had 33 students, was viewed by the teachers as a quite special class. They were above average achievers in mathematics who had a great concern with their school marks in mathematics, having in mind their academic future, namely their access to university. Good results in tests and examinations were some of their utmost objectives.

These students were used to work hard and were persistent. They always made their homework and some of them even gave the impression of having private lessons in extra school time.

Resources and class organization. The computer activities were performed during class time and took place in the computer centre where six computers and two printers were available. As the number of students was quite large, they were divided in eleven groups of three. Six groups were working with the computers and five were doing parallel activities without them. On the subsequent lesson groups would interchange roles and as soon as they finished both kinds of tasks there would be a discussion with all the class to expose the most important conclusions and results. Occasionally one data-show was used to help this synthesis of students' findings with the computer.

The software used throughout the experience was an electronic spreadsheet, the SUPERCALC 4. The teachers chose it having in mind their small experience with computers and the goals they pursued. They examined some other computer graphing utilities as alternative options and decided that the spreadsheet was quite easy to manipulate with a minimum of training. Furthermore, they judged it as an appropriate software to deal with algebraic expressions, calling for a decisive intervention of the students in organizing and creating tables and graphs.

According to their reports, students were reasonably capable of interacting with the spreadsheet after one introductory lesson of fifty minutes.

As the teachers described a typical lesson with computers, students were indicated their exact seats in the beginning of the class and had already on their tables a worksheet that would guide their investigations with and without the spreadsheet. They would start working in their groups and the teachers (usually the five of them) would circulate, monitoring and observing the students' progresses. They would answer students' questions, having always in mind never to give the solutions they were looking for. Instead, they would pay attention to them, making suggestions or pointing out possible clues, but sometimes they would just encourage students to think a little more.

After a sequence of classes with computers, there would be a summary lesson with the teachers leading the discussion. Students were asked to present written reports on their work and these were given to the teachers before the plenary lesson. Some of the issues brought to this large group discussion came from such students' reports.

The computer activities. Computers were associated with three main topics of the national 10th grade curriculum: (a) domains of rational and algebraic expressions, (b) polynomial equations and inequalities, including second degree expressions, (c) algebraic expressions representing straight lines, including algebraic conditions for parallel and perpendicular lines. All of these topics came at a different time. The first was in November, taking about two weeks; the second was in February/March and lasted for a month; the third was in May/June and took two weeks.

The teachers recognize the difficulties they had in creating good problem solving activities allowing the students the desired freedom in their own computer experimentations. They also admit their failure in the design of some of them and in their efforts to turn them into less directive formats.

A brief analysis over some examples of the worksheets, may contribute for a clearer idea of their nature. Starting with the first topic, domains of rational and algebraic expressions, the activities were aimed to illustrate how the spreadsheet "detected the forbidden values". In fact, the spreadsheet was used to reveal error messages for some values of the variable. Students introduced formulas in some cells as translations of the expressions they were asked to analyse. Then, they freely assigned values to the variable and observed the results obtained with each formula. Nothing was said in the worksheet about ranges of values to consider and students had to make many trials. Some of them used one single cell to allocate the values of the variable and others conducted a more systematic procedure by means of creating a column of values for the variable. In this way they generated a table to have a better view of what was happening to the transformation of a set of values. The students did not feel the need to explore any possible graphs for their analysis. With this kind of activity, students became aware of the meaning of error messages in such cases as the assignment of zero to a denominator or negative values under a square root. The students began to think in terms of finding the values that did not belong to the domain and, from there, they were able to get the desired domain.

The second set of activities concerned solving quadratic equations and inequalities and the teachers found in it a good starting point to study the parabola in its several aspects: zeros,

extrema, monotony in intervals, upwards and downwards concavities, etc. All this was sustained by the analysis of graphs created with the spreadsheet. The first approach to the understanding of different expressions for different parabolas was based on a worksheet where students had six graphs selected by the teachers to represent typical examples. They expected to have covered all the possible cases with those few typified examples and hoped that students would be capable of making correct generalizations from that. Instead, students were driven to false generalizations due to unforeseen limitations of the material provided. They concentrated their attention on certain details of the parameters and related them to some irrelevant aspects of the parabolas, therefore jumping to conclusions that were definitely uncorrect. The teachers next expedient was to create a simple spreadsheet file so that students only needed to introduce and modify the values to assign to each parameter of a quadratic expression in order to visualize the effects produced in the consequent graphs. This was seen as a much better strategy to make students understand the relation between the coefficients in the expression and the type of parabola generated. Later on, students also learnt to solve quadratic inequalities in a non standard way, that is, making the immediate sketch of the appropriate parabola.

Other activities included in this sequence of lessons were of a different nature and involved a problem solving task. Students had to discover the right value for the length of an edge of the square face of a box so that the box had its maximum volume. This problem and a similar one were solved in the spreadsheet and were a motivation to study polynomial inequalities of third degree, using graphs as a resource.

Finally, for the third topic, the activities were, once again, devoted to guide students in investigative processes. On the study of straight lines from an algebraic point of view, students used a spreadsheet file previously prepared by the teachers. They looked for the effects of changing the parameters in a linear expression over the position of the straight lines shown in the graphs. Students had to find the conditions for parallel and perpendicular straight lines, as well as to relate the coefficients with their slope and points of intersection with the axes.

As teachers pointed out, computer activities were intended to develop students' awareness of graphical information and to support new strategies for the solution of algebraic problems traditionally treated in formal ways requiring paper and pencil computations.

Students' reactions. The reactions of the students to the introduction of computers in the classroom were not uniform. There were clearly two major kinds of attitudes expressed by them. Some accepted the idea very well, apparently enjoyed their work and even found, in dealing with computers, ways of showing hidden capacities of intuition, discussion of ideas, conjecturing and reasoning. As the teachers pointed out, those were the students who had already manifested in class a flexibility to deal with more open questions. Moreover, those students were not the most brilliant ones that usually got everything correct in their written tests.

On the other hand, many other students were very suspicious about the computer activities suggested by the teachers. In the beginning, they showed a cooperative attitude, having in mind

to please and even to help the practising teachers, assuming that this experience was a constraint of their own training program. But after some time, they started manifesting a clear rejection of the work being developed with computers, claiming that they needed a different type of teaching, a more serious one, in order to be prepared for future examinations. They wanted more practical exercises, more teacher exposition and more individual work in the classroom. Some of the students demonstrated they were not prepared for achieving their goals in the tasks involving computers. Their failure brought out some revolted feelings from the moment they realized they were having troubles in keeping up with their standards of academic achievement in mathematics.

Things did not go very smoothly in terms of evaluation either. Indeed, teachers explained a certain uneasiness in conciliating the kind of performance students revealed in their work on computers with the difficulties they showed later on, when asked to elaborate on some questions in a more consistent and rigorous form, without the computers. They also mentioned the confusing way of students expressed themselves in a written test situation.

Although a few students got very interested in going beyond the objectives of some activities — as it happened with the domains of algebraic expressions when a group of students decided to investigate the sense of dividing by zero and shared their discoveries with the class — the students' reports on their work were very disappointing. They turned out to be a strict list of their answers to the questions from the worksheets.

The supervisor teacher who is this year teaching the same class (now on 11th grade) showed her sorrow about the fact that students are not interested in taking again the possibility of working with computers in mathematics. She assured that students seem to be more cooperative and more likely to question their ideas but still persist on asking for detailed explanations from the teacher and valuing their academic results above everything. In a word, they continue to be quite reluctant to accept deep changes in their structured way of learning mathematics.

Teachers' reactions. For the teachers, and overall, this was a very positive and rewarding experience. They enjoyed working together, raising questions about current content and methods of teaching mathematics. They became very sensitive about the power of graphical representations for mathematical concepts, namely for functional ideas. The four practising teachers discovered the computer as an educational tool and became much more articulate about their sought goals and more aware of possible pitfalls in their approaches. The teachers constituted a group with an excellent rapport. They had lively discussions in which they planed work and produced proposals and materials. Their personal and working relationship with the supervisor was also very good.

As the experience went along, the teachers felt the need to explain to the students what was the purpose and the meaning of the proposed activities, in terms of their mathematical experience, but they found difficulties in having their messages accepted.

The teachers commented on the huge effort they had to do to make students communicate and share their ideas, and in the end they expressed some doubts about having succeeded on it.

However, they were confident that some positive changes were induced in their students and pointed that these are hard phenomena to detect and need time to be revealed.

They were concerned with the poor nature of the reports produced by the students. They explained it by the insufficient orientation and lack of explicitness they provided for their organization as it was an activity that students were not used to produce in mathematics.

In spite of the obstacles they found, the practising teachers felt that the experience changed their own way of seeing mathematics and became even more convinced that experimentation is an important feature in the process of doing mathematics.

Questions

This innovative process rises a number of issues that certainly deserve an in-depth discussion. It is impossible to state with absolute confidence what went right and what went wrong and we will not assume the role of judging the teachers. Rather, in our approach, we will try to present what appears to be the ways in which both the teachers and ourselves try to make sense out of what happened in this experience.

Different perspectives between teachers and students. It is quite apparent that different perspectives about the value of the computer-based activities developed in the teachers and in the students during this activity. As the experience proceeded the teachers got more and more excited about the computer and discovered many of its possibilities for the exploration of mathematical concepts and supporting problem solving activities. The students, or at least many of them, increasingly found the computers an embarrassment, a device that, instead of promoting, hindered "real learning".

As the teachers perceived that the reactions of the students become not very favorable to the proposed activities, they developed possible explanations for this phenomenon. It resulted, in their opinion, of a multitude of factors, all of them stressing the special nature of this 10th grade class:

—The students' former learning habits and assumed roles in the discipline of mathematics; this let them to feel uncomfortable with classroom work that required a lot of discussion;

—Their lack of familiarity with this kind of activities; it is something very different of what they have in mathematics classes and in their home study as they get prepared for the tests;

—Their strong competitive spirit; the students were used to compete against each other for the highest marks in the tests and courses and not to cooperate (this competitive spirit has originated discipline problems with other teachers);

—Their great concern in getting good grades and the fear that this kind of activities made their performance decrease; as university entrance partially depends on these grades this could threaten some of their personal expectations;

—Their need for feeling of success in their work; the students appeared to need clear feedback mechanisms on the success of their work and felt uncomfortable as these mechanisms were not clear or pointed towards increasing failures;

—Their cognitive styles, which could be classified in two different kinds — the analytical and the intuitive; the analytical students were used to work with formalised mathematics and had the greatest difficulties in the proposed tasks; the intuitive adjusted themselves better to the computer work, although they still had difficulties in integrating this learning in more formalised tasks;

—The special arrangement for this class, with 4 practising teachers, plus the supervisor teacher, sharing responsibilities, teaching in a rotative way a sequence of lessons, created an ambiance in which students lacked reference points.

It should be noted, however, that all these reasons pointed to preexisting factors, and do not include things that were done in this experience.

Nature of the proposed activities. The kind of activities that were proposed to the students may have promoted in unexpected ways their negative reactions. The teachers themselves felt uneasy with some of them and considered that they should be improved. Some aspects in which this might be done include for example (and the teachers agree with these comments):

—A broader exploration in the case of domains of expressions; the activity carried was quite narrow, following a strictly algebraic perspective (with no functional or graphical ideas), well in the spirit of the current Portuguese syllabus;

—A more gradual exploration of the second degree polynomial; the activity as carried begun with a complex task which was quite difficult to handle; the students were required to make some generalizations from this situation, but they did the "wrong" generalizations, which were false, and there was no clear way for them to proceed correcting their conjectures;

However, there is something more difficult to describe about the tasks that might also be of importance. The proposed activities do not provide the sort of "closure" that most teachers feel necessary to give confidence to the learner. After doing them the students were left with some feeling of dispersion, of lack of structure, which may have contributed to their anxiety in a very significant way.

What Else? Some other factors, besides the current attitudes of the students and the design of the activities may have contributed to the verified mismatch between the views of teachers and students. They have to do with the pedagogical relation and the classroom climate. We mention very tentatively:

—The way teachers decided to respond to the questions of the students; to require the students to struggle with the things that they do not understand may appear to be a good instructional practice to arrive at but not be a good strategy to start with — it may inhibit the establishment of an affective relationship between teachers and students without which the students do not respond effectively to the teachers' demands;

—The lack of a clear attempt to integrate the students as responsible partners in this experience;

—The organization of the classroom in two main groups, one on and the other off the computer, taking turns, which has worked well in previous experiences with younger students, may have promoted a pace of work that is not suitable for students at this grade level and with these expectations;

—The fact that no way was found to overcome the special situation of this class (the four practising teachers plus the advisory teacher), providing strong reference points to the students;

—Eventually there was an insufficient conceptualization of the experience: What are our objectives? Why do we want the computer? What should be its role? What is the role of the students? What shall we do to let them assume it?

—The lack of concern with possible limitations or undesirable features of the software; the spreadsheet — as any other general purpose program — has limitations as well as potentialities and it is necessary to be aware of them to create the appropriate educational activities and learning environment.

In fact much as been said about the potentialities of the spreadsheet. Let us mention just a few limitations: (a) there is only one way for reasoning to proceed: from a formula (or given values) to a table to a graph; (b) "misbehaved" points may exist in the domain of a function and the spreadsheet does not indicate them (specially if they do not appear on the constructed table); and (c) non required lines may appear on the graphs such as pseudo-asymptotes and other connecting lines.

What did the teachers learn in this process? For the teachers this was a strong and positive experience that they enjoyed. It will certainly leave a mark for many years in their professional styles. However, the teachers were aware that their experience had several limitations. They were not sure how to regard the difficulties of the students, and specially how to overcome them. They and we can draw several lessons from what happened (hopefully these will not appear as contradictory):

—Innovation is not always easy to implement; sometimes things develop in unexpected ways;

—Innovations may result or not result; there is not a special axiom saying that all the innovations are necessarily successful as we first conceived them;

—Innovations are not valuable just by themselves; their value depends on the measure they contribute towards students' learning and growth;

—The fact that things become difficult is not a reason for giving up; it is a reason to let us think on the sources of problems and on how to overcome them;

—Group dynamics is a factor of creativity and personal confidence in carrying out an innovation project; it brings a completely different experience from the teacher working in isolation;

—The spreadsheet, with all its limitations, can be indeed a very useful tool for mathematics teaching and to promote a different approach to many mathematics topics;

—There are many ways of working in mathematics and of what means to learn mathematics;

—The activities usually made in the classroom tend to leave out some students which do not adapt themselves easily to formalised mathematics but can have success with different approaches;

—Also, students with facility in formalised mathematics may have great difficulties in investigational tasks or in applying mathematics to real world situations;

—Conceptions and deeply established habits in the students may pose quite difficult problems to innovation processes;

That these lessons were taken seriously is best shown by the fact that this year one of the practising teachers, now in a different school, is doing a similar experience, but with improved activities. The supervisor and another teacher are working on innovative activities for 9th grade geometry.

Conclusions

The students started as being cooperative and sympathetic towards the experience. They gradually changed their attitudes as they felt threatened in their expectations and securities. These attitudes and the factors that underline them should be viewed as a given that needs to be taken into account in an innovation process instead of an explanation for its difficulties.

We will end with three further conclusions:

—In innovation processes the desire of innovate and the willingness to make mistakes and run risks are not sufficient; it is necessary to complement them with hard work of conceptualization of objectives and strategies and reflection on the outcomes;

—New and promising ideas such as the computer, problem solving, investigational tasks and other pedagogical proposals are always multi-sided and may carry with them unexpected issues to which attention should be paid;

—The pedagogical relationship which is established between teachers and students is always a key factor in the learning process, and is a particularly critical condition for the development of an innovation.

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