1993

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# Necessary research in mathematical modelling and applications

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## SUMMARY

Curriculum development in mathematical modelling and applications is an active area of work in mathematics education. However, it needs to be more closely connected to research. This paper suggests five different areas in which research may be important: epistemological, cultural and social issues; students' cognitive processes; curriculum approaches; didactical strategies and classroom processes; teacher development. Some examples of recent and ongoing studies are provided. The paper concludes by presenting a few remarks about the nature of the necessary research, and sketching some ways in which this effort can be undertaken.

## 1 INTRODUCTION: NECESSARY RESEARCH?

Just by itself, or in association with computers, mathematics plays an increasing rôle in modern society. It is used to convey ideas, to support controversial arguments, to control information flows, and to assist in problem solving. For such purposes it may be viewed as a descriptive language, a classificatory instrument, an encoding mechanism, or a decision-making tool. This implies that the ability to use mathematics in concrete situations, and to analyse critically

how it has been used by others, become in themselves important educational objectives.

In most countries, applications of mathematics and modelling have a secondary, if not negligible, place in the school curriculum. To reverse this situation, it will be necessary – amongst other things – to undertake considerable work in terms of curriculum development and research. Curriculum development includes the production of well-articulated proposals, supporting materials, classroom activities, teaching modules, and integrated courses aimed at these educational objectives. Research needs to address all the problems that emerge in implementing these activities at the national, school, and classroom level, and to suggest strategies for overcoming the inevitable difficulties.

Curriculum development is already taking place for in several countries, and there is a considerable bibliography in this area (see Blum, 1994). However, there are some inherent weaknesses in this effort – a critical one concerns its insufficient research base. This paper indicates some of the areas in which research is most clearly needed. When appropriate, examples of recent or ongoing studies are provided.

## 2 GENERAL AND EPISTEMOLOGICAL ISSUES

One important area for research is the nature of mathematical models and the epistemological, cultural and social status. There are models with quite diverse characteristics, purposes and functions; however, there is a shortage of literature that adequately deals with this matter.

The significant work of Davis (1988), Skovsmose (1988), and Booss-Bavnbek (1991) point to some critical features of misusing mathematical models. Other kinds of analysis, emphasising historical and mathematical dimensions, are necessary to yield constructive orientations for curriculum decision-making and for classroom teaching.

Consider, for example, the common occurrence of dynamic systems in the educational literature on modelling. Indisputably, they have very interesting properties and find many applications, but that is not a reason to downplay the importance of other kinds of models. We need, therefore, a comprehensive taxonomy of models, taking into account their nature and relevance, enabling a better informed choice of curriculum options and of the corresponding implications.



## 3 STUDENTS' COGNITIVE PROCESSES

The cognitive processes of students, when they work in modelling activities and in applications of mathematics, constitute an important area of research which is already in progress.

Boero (1992), drawing on extended experience from a long-term development project, claims that *complexity* in applied mathematical problems concerns three distinct aspects,

- the number of choices and steps that the pupil must keep under control,
- the coordination of different concepts and procedures,
- the selection of pertinent aspects in a 'complex' reality.

A framework for the cognitive processes involved in model formulation was proposed by Lambert, Steward, Manklelow, and Robson (1989). It was set at three levels,

- beliefs, including other knowledge and fundamental assumptions,
- metacognition, that is monitoring, evaluation, planning and control,
- reasoning within the mathematical domain and reasoning within the problem domain.

However, further work is necessary to ascertain the relationships between all these elements.

Lesh (1982) and Lesh and Kaput (1988) also produced an interesting theorisation about such processes, based in what is called 'local conceptual models'. According to this, related questions are

- to identify the nature of the primitive conceptual models of the students for different mathematical ideas,
- to explain how the deficiencies of the diverse models are detected,
- to explain how the most successful models are gradually constructed,
- to describe the cognitive characteristics associated with the use of unstable conceptual models, and how their negative influences can be minimised.

This research concerns students' work on problems that are interesting and realistic, but mathematically quite simple. Will such observations have similar

importance for other kinds of problems?

Reasoning in situations involving data gathering as part of the modelling process was investigated by Hancock and Kaput (1990). They contend that, for young students the iterative process of data modelling is essentially "a tangle of loops small and large", whereas for experienced data modellers it comes closer to "a grand loop from data design to data collection to data analysis and back to the beginning". This is also a line of work that deserves to be continued.

There are many other issues relative to cognitive processes that need to be considered.

- In what specific aspects are the processes used by the students in applied situations different from those that they use in current mathematical problems?

- What are their greatest difficulties in dealing with applied situations? In what way will their general conceptions and attitudes towards mathematics tend to interfere with the realisation of modelling activities?

- Can the students adopt the modelling cycle (Kerr and Maki, 1979; Niss, 1989) as a fundamental conceptual reference? In what parts of this cycle have they greatest difficulties? What heuristics relative to the global process, or relative to each phase may assist the students in solving applied mathematical problems?

## 4 CURRICULUM

We also need research on the curriculum itself. The introduction of applied-mathematical problems and modelling tasks presupposes a clear indication of objectives and strategies, and an articulation with the remaining content, objectives, and curriculum activities. As this is virtually a new strand to be introduced in the teaching-learning process, we need to study the related implementation problems carefully.

Modelling and applications may appear in the curriculum in several ways. They may constitute

- the fundamental subject, which gives the structure to the course (as [illegible] by Garfunkel, 1994)



- activities that run in parallel with a programme organised around established mathematical topics,
- mostly starting points for the introduction of new topics.

The curriculum may stress

- extended projects, that take weeks or months to complete,
- situations that may require one or two classes,
- more specific activities, several of which can be completed in a single class-period.

Furthermore the curriculum may emphasise the construction of models, or the use and exploration of models already constructed. It may, finally, consider situations taken from physics, biology, economy, or ones from modern life that are not clearly related to any specific subject matter area.

Van Streun (1990) compared three kinds of curriculum, all giving importance to applied problems. The first offered just mathematical and applied problems, the second gave implicit attention to heuristic methods and little formulation of mathematical concepts and techniques, and the third gave explicit attention to heuristic methods and more attention to mathematical concepts and procedures. He concluded that this last approach led to superior cognitive schema.

With a different focus, some important work in this area was also carried out by Kaiser-Messmer (1991), who indicated different orientations that may inform a curriculum

- stress in utilitarian and pragmatic activities,
- emphasis in scientific and humanistic educational goals,
- an integrated approach.

This kind of work must be continued, contrasting experiences realised in different countries, and discussing questions such as the following.

- What are the implication of each curriculum option? In which circumstances do they work satisfactorily?

- How to articulate the use of real-life situations as a starting point for concept learning (and learning of mathematics content in general), and as problem situations that must be studied in depth in themselves?

- What is the didactic rôle of the more ambitious projects and of the shorter activities? When must each one be used, and in what conditions?

- How do we establish different kinds of real-life situations, related to different underlying models, with different pedagogical rôles?

## 5 DIDACTICAL STRATEGIES AND CLASSROOM PROCESSES

We now have the question of didactical strategies at the classroom level. Broad curricular orientations still give room for very different possibilities – some very likely better than others – when one comes to the teacher-student interaction. Unfortunately, research conducted in real classroom settings tends to be very scarce – for an exception, see Abrantes (1991).

To understand the difficulties of the teachers, and to be able to give them adequate support, this is a most necessary area of work. Significant questions are, for example, the following.

- How to help the students to become more competent in solving applied mathematical problems? In what ways may the students be supported by the teacher? What pedagogical principles must be considered? What are adequate forms of intervention?

- What grouping strategies are more adequate for each kind of classroom activity – individual work, group work, mixed?

- What tasks must be done in the classroom, and what must be done outside the classroom – and how?

One further question, that inevitably arises in any curricular and didactical option that embodies a truly innovative approach, is how to carry out the assessment of the students (de Lange, 1987; Leal, 1992).

## 5 TEACHER DEVELOPMENT

Finally, it is necessary to undertake research about the teachers and the teachers' development processes. Teachers are not just obstacles to innovations. They have their own rationales, constraints, and resources. They come into teaching



with distinct training, motivations, and competence. With respect to the relation of mathematics and reality we need to address questions such as the following.

- What difficulties have teachers in understanding the nature and rôle of applications and modelling?

- What difficulties do they have in terms of carrying out these activities in their educational practice?

- How do teachers tend to use different sorts of curriculum materials to support applications and modelling?

- What kinds of professional development programmes may be developed to help overcome their difficulties and insecurity?

Very often primary and secondary school teachers, and others as well, think that research has very little to do with them – it is something to be carried out by others, in higher education. That is a too restrictive view of research, that pays lip service to the needs of the teachers themselves. I would like to propose that there should be a close connection between curriculum development and research, or at least some kinds of research.

## 7 CONCLUSION

All these research areas are closely interrelated, interacting with each other. All of them need an urgent attack. We must have theoretical analyses – to clarify concepts and define fundamental questions, to help to organise conceptually this still messy domain. We also need empirical research – that contrasts the different proposals with the realities of practice, and helps to distinguish what works (at least in each particular case) from what looks interesting but does not work so well.

Research, here, is essentially the search for answers to well-posed questions by rigorous, systematic and verifiable processes. There are certain kinds of research that require a very heavy instrumental and conceptual machinery. Of course, some attention is needed not to vulgarise the term research, as is sometimes done. However, it should be recognised that there are simpler forms of research, which imply far fewer resources and which and are still suitable to address the problems at hand. At any rate, much is to be gained if we can connect more

closely research with development, and school teachers with researchers and teacher educators, in the frame of well-established projects.

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