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CHAPTER 6
A COLLABORATIVE PROJECT USING NARRATIVES:

What Happens when Pupils Work on Mathematical Investigations? [1]

Abstract. Mathematical investigations involve searching for patterns, formulating, testing, and justifying conjectures, reflecting, and generalising. Doing investigations in the classroom is a powerful activity for students' learning but poses many challenges to the teacher. To study the professional knowledge involved in this kind of work was the aim of a collaborative action-research project that involved one classroom teacher and two university teacher educators. We used narratives to depict relevant elements of teachers' activity and to show key aspects of their dilemmas and uncertainties. This paper discusses the role of the collaborative work undertaken by the participants in the project as they reflected about classroom practices and curriculum issues, based on a narrative of a class where sixth grade students worked on a mathematical investigation.

MATHEMATICAL INVESTIGATIONS

Mathematics has several faces. It is a body of knowledge, but it is also a human activity, a language, and a tool to deal with many kinds of problems. Much more than knowing how to do algorithms and procedures, pupils must show intellectual flexibility, capacity to deal with different representations, formulating problems, modelling situations, and evaluating the results (MSEB, 1989). Mathematics learning, thus, needs to include opportunities for pupils to get involved in genuine mathematical activity. Instead of presenting mathematics as a finished product, beginning with definitions and statements to go to examples and exercises, teachers may emphasise its development processes, starting with questions and issues, and showing how it is at the same time "an experimental and deductive science" (Pólya, 1945, p. vii). The social processes of negotiation of mathematical meaning that occur in the classroom (Bishop & Goffree, 1986) parallel the processes that dictate the acceptance or rejection of a mathematical concept in the research community. Mathematics is a social construction and, therefore, it is impregnated with values like any another product of human thought. To provide pupils with this sort of experience, we need to bring their activity close to the activity of the mathematician, transforming the classroom in a small mathematical community (Schoenfeld, 1992).

[1] Ponte, J. P., Segurado, I., & Oliveira, H. (2003). A collaborative project using narratives: What happens when pupils work on mathematical investigations? In A. Perter-Koop, V. Santos-Wagner, C. Breen, & A. Begg (Eds.), *Collaboration in teacher education: Examples from the context of mathematics education* (pp. 85-97). Dordrecht: Kluwer.

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The classroom activity depends largely on the nature of the mathematical tasks and on the classroom organisation set up by the teacher. Classes where pupils work on extended investigations and projects, work together in small groups, and get involved in collective discussions and classes where they just do simple exercises on their own and listen to the teacher cannot run in a similar way. The classroom activity is related to the nature of the learning environment and the classroom culture and is heavily influenced by how the teacher introduces the different tasks and supports pupils working on them. Of course, many other factors contribute to the classroom activity, including some related to pupils, notably their conceptions and attitudes regarding mathematics, their previous knowledge and experience on mathematical work and, more generally, their relation with the school. Other factors include school organisation and ethos and parents' culture, resources, and expectations. This paper focuses on the nature of the tasks and the aspects of the learning environment that are directly amenable to teacher intervention.

Mathematical tasks in which pupils get involved – problems, investigations, exercises, projects, constructions, productions, written reports, essays, etc. – provide the starting point for the development of their mathematical activity. They must awake curiosity and enthusiasm, appeal to pupils' knowledge, and promote the development of new concepts and ideas. Tasks can be defined by the pupils themselves, but are, most of the time, proposed by the teacher; in any case, tasks are interpreted by pupils and can originate very different activities (or no activity at all), depending on their disposition and the classroom learning environment (Christiansen & Walther, 1986).

This action-research project focused on pupils' investigations. These are tasks intended to promote mathematical processes such as to look for regularities, to formulate, test, justify and prove conjectures, and to reflect and generalise. Investigations are "open situations" (sometimes also called "open-ended problems"), that may be set up in a variety of mathematical and real life contexts. Their point of departure may be a question proposed by the teacher or by a pupil.

For a pupil, an investigation may constitute a motivating and challenging activity. As, in any genuine mathematical problem, pupils do not have immediately accessible a way of solving it. In fact, they often need to reframe the question in their own terms to start doing some productive work. A mathematical investigation requires that pupils justify and prove their statements mathematically and present their arguments to their colleagues and to the teacher, which are important competencies in mathematics education (NCTM, 2000). As pupils discuss their different conjectures and justifications, they work in class as a small mathematical community engaged in the production of mathematical knowledge.

For a teacher, this kind of work also poses deep challenges. An extensive planning is required. The selection or creation of tasks, aiming at different educational objectives, needs to take into account the specificity of the class and its history. Doing it, the teacher acts as a "curriculum maker", delineating objectives, methodologies and strategies, and reformulating them according to his or her reflection on practice. Both the creation and the reformulation of the tasks consume time and demand an investigative attitude. After having selected the situation to

consider, the teacher has to so some further planning, including taking decisions regarding the organisation and management of the class. Are pupils going to work individually or in groups? How to constitute the groups? Should time be provided for some all class work? Such decisions are critical regarding the nature of the learning environment. They depend on the task but also on the educational objectives established by the teacher. Another issue is to foresee the time needed for the activity. It will be possible to carry through an investigation in only one lesson? For how much time the pupils will likely be interested in the activity?

Good tasks are an essential ingredient in a mathematics classroom but it is also necessary to consider what teachers do, the questions they make and the interactions they promote. If classrooms are to become mathematical communities, interactions among pupils become essential. Small group work may encourage pupils to share ideas and explain their approaches. Discussions involving the whole class may favour the development of the ability to argue and to communicate mathematically.

The work in an investigation develops usually in three main phases that may extend by one or more class periods:

- *Start.* The task is introduced by the teacher and the pupils begin working on it, interpreting the situation and considering strategies to follow;
- *Development.* The task is carried out by pupils, who work individually or in small groups, and the teacher interacts with them;
- *Summing up.* The results are presented by the pupils and discussed by the whole class.

The way the teacher presents the task is very important. A question, just by itself, cannot generate any investigation. As Mason (1991, 16) puts it: "A question is just words with a question mark". It is impossible to anticipate all the reactions of pupils. Once the activity begins, the support to give pupils, helping them to overcome certain difficulties is another rather complex aspect of the role of the teacher. Some support has to be granted, but not too much nor too little. The final discussion regarding the work done by pupils is another critical stage. Without such discussion the value of the activity can easily be lost (Cockcroft, 1982). This is the moment to consider the strategies, hypotheses and justifications provided by different pupils or groups of pupils, with the teacher acting as a moderator. The teacher tries to bring to the attention of the group the most important aspects of the work they did and stimulates pupils to question the assertions of their classmates. Thus, the development of pupils' competence to communicate and argue mathematically are two important objectives in this phase of the activity.

To investigate the challenges to teachers' professional knowledge posed by this kind of classroom activity was the main goal of the project. It was developed as an action-research project based in the co-operation of two teacher educators and a sixth grade teacher which we describe in the next section.

COLLABORATION AND NARRATIVES

As participants in this project, we were interested in exploring—in a collaborative way – the possibilities of pupil's mathematical investigations and of narratives in educational research and in teacher education. We take collaboration as representing an activity carried out by a group of people with common objectives who jointly negotiate their working processes. It may involve partners with similar or different backgrounds and professional roles but necessarily requires the joint construction of a common ground—shared objectives and working processes.

Collaborative research may be very useful to study some kinds of problems – specially those problems that hardly can be studied by isolated researchers or by research groups whose members do not hold all the necessary competencies. Many classroom phenomena enter into this category. The study of questions about classroom dynamics and teachers' professional knowledge requires the active involvement of teachers committed to a deep analysis about their own practices as well as of researchers interested in teaching. The point of view of practitioners in the study of professional practice is essential to know what enhances students' learning (Bednarz, Desgagné, Couture, Lebuis, & Poirier, 1999). It also requires deep involvement of researchers with experience in defining research questions, instruments, and procedures for data collection and analysis.

Collaborative research, besides being very useful to study complex phenomena, may also be of essential value to promote the personal and professional development of all those involved in it. Different people, interacting with each other for an extent period of time in a common endeavour, besides accomplishing a specific task, may learn a lot about different viewpoints, different concerns, and different working methods, and even about themselves. Collaborative activities allow for the mutual influencing of different perspectives—each one informing and transforming the other (Olson, 1997).

This work may draw on the specific competencies of all partners involved, but also needs to pay attention to the creation of common objectives and appropriate working procedures that help everyone to make a strong contribution to the development of the task. In a collaborative activity, different participants need to share a common aim, but may have rather different immediate goals. When teachers and university researchers are involved, it is natural that the teachers will be primarily interested in developing knowledge to improve their practice and researchers in developing knowledge of interest for the scientific community (Kapuscinski, 1997).

Collaboration does not mean necessarily that everyone has the same power and the same role. Absolute mutuality is rarely achieved. What is critical is that all participants feel comfortable in their roles and are attentive to the needs of the others and open to negotiate the understandings that emerge from the collaborative effort (Castle, 1997). This is not an easy process. However, tensions that arise in collaborative relationships may help to keep these relationships alive and dynamic. In collaborative processes, there are no easy and safe answers. But what is problematic may provide the momentum for further learning as each partner tries to understand him or herself and the others (Olson, 1997).

A COLLABORATIVE PROJECT: INVESTIGATIONS

In this project, our team worked together for a long period (about four years). There was a joint theoretical work discussing texts about mathematical investigations, classroom dynamics and narratives[2]. We also set up a collection of tasks and discussed the structure of a class with students working on investigations. We paid special attention towards the nature of teacher-student interactions and the role of classroom discussions.

The general framework for the investigation classes and the specific tasks to be offered to students were developed collaboratively. Things to do were decided in joint meetings and products of the project were thoroughly discussed so that they would reasonably satisfy all project members. The specific preparation for the class, involving the choice of day to carry it out, the organisation of students, and the form of presenting and conducting the task were mainly the decision of the teacher.

The process of knowledge construction in this project was based in the elaboration and analysis of narratives about situations occurred in classes where pupils were working in mathematical investigations. It was thought that these narratives would indicate aspects of dilemmas and uncertainties of the teachers and evidence elements of their professional knowledge in this type of educational activity.

Narrative analysis, as a method of educational investigation, is attracting increasing attention. We briefly refer the main ideas that made us to consider them in this project. We view a narrative or story as a way of telling a sequence of events with three basic elements: (i) a situation involving some conflict or difficulty, (ii) one or more characters who get involved in the situation with given intentions, and (iii) an ordered sequence of events deciding the conflict in some way. In other words, a story contains reference to people, places, and events fitted in an ordered sequence that implicitly suggests some causality. Every human being is a storyteller, seeing the present evolving from the past and directing towards the future. An episode of someone's experience is a narrative unit if it brings sense and unity to that experience (Carter, 1993; Clandinin & Connelly, 1991; Connelly & Clandinin, 1986). Stories constitute an integral part of our daily experience. A basic idea is that we use them to organise our experiences of social interaction. According to Bruner (1991), we organise our experience and our memory of human events in the form of stories, that is, they are phenomena of our natural thought. We live through stories, that is, we think, perceive, imagine, and make moral choices according to narrative structures. The creation of stories allows us to impose order and coherence in our experience of the real world events (Carter, 1993).

Another basic idea is that narratives constitute a way of knowing particularly related to action. Stories are ways of knowledge emerging from action. They are "concerned with the explication of human intentions in the context of action" (Bruner 1985, in Carter, 1993, p. 6). Stories, with their multiplicity of meanings, are a form particularly adjusted to express knowledge associated with the complexity of action. Since teaching is an intentional action in a situation, much of the essential knowledge that the teachers have about teaching comes from practice, that is, from acting as teachers in classrooms. Thus, to understand the thoughts of the teacher, we can start by looking for those stories that structure the way this teacher thinks about

the events of the classroom (his or her practical theories). However, we must note that, in their narratives, teachers do not just remember and tell their experiences, they also recreate their own stories, reconstruct meanings, and redefine their personal and professional self (Cortazzi, 1993).

A key idea in this project is that the production of narratives is a way of promoting the collaboration between teachers and teacher educators. The narratives were drawn from episodes occurring in classes conducted by the teacher in the project. The relationship established among participants as they jointly construct narratives, foments the reflection on practice and allows a deeper understanding of eventual changes occurring in that practice.

The general method of narrative research consists of understanding and reconstructing, in extended reflections involving the participants, the narrative units of their stories. Narrative research tends to start without a pre-specified problem, but with an interest in a phenomenon that can be understood in a narrative way (Connelly & Clandinin, 1986). The writing of a narrative is the first step of the interpretation. The observation and the joint reflection on lived situations play, in this step, a basic role. The analysis is a second step. For Labov (quoted in Riessman, 1993), a narrative can be decomposed in 6 basic elements: (i) abstract (summary of the substance of the narrative); (ii) orientation (time, place, situation, participants); (iii) complication (what happened); (iv) evaluation (the meaning of the action, the attitude of the narrator); (v) resolution (what finally happened); and (vi) *coda* (returns to the present perspective). In its final form, the narrative continues open to new readings and constructions. A narrative carries a strong cultural and historical load. The truths that we construct are significant for specific interpretative communities in well-defined historical circumstances. Each level of the model involves a reduction, but also an expansion: the accounts tell aspects of global experience but also join other interpretative elements. The analysis of a narrative implies to select, to point out, to relate and to compare. As in all research processes, it is a key creative moment. One intends that the analysis will not corrupt the voice and meaning of the practitioners, but enrich and clarify it using the multiplicity of experiences and perspectives of the members of the project team.

Next, we present a narrative written originally by the middle school teacher in the project, Maria Irene Segurado. This is not the original text, but a refined form after several stages of discussion among the three of us.

AND WHEN PUPILS FOLLOW UNEXPECTED WAYS? ...

It was just another Wednesday. However, I felt anxious with the lesson that I was about to begin with my sixth graders. I had great expectations. The task that I had prepared seemed to be quite challenging and, given my knowledge of the pupils, I foresaw that they would feel the same pleasure I had, in the eve, exploring it.

The task, named *Explorations with numbers*, asked the pupils to discover relations between the numbers in the figure and to record their conclusions:

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
...	...	...	...

In the beginning, I gave the pupils small 'tips' about what they could observe (regularities, the behaviour of multiples, divisors, prime numbers, perfect squares...) and all the groups started to work very actively. It was not the first time that they were working in an investigation and they had no problem in understanding what I intended. I was constantly requested by the groups, not to answer questions but rather to see their discoveries (secretly, so that the other groups would not hear, ruining their 'show' in the final discussion). Some discoveries readily appeared:

- *The numbers in the diagonals, from right for the left, grow 3 to 3 units, from left to right, 5 to 5 units.*
- *The 2s table is in the first and the third columns.*
- *The 6s table is in the first and the third columns jumping two numbers.*
- *Prime numbers are in the odd columns, but strangely 2 is in an even column.*

Suddenly, the group that included Bruno, Ricardo, Cândido and Pedro called me, showing great enthusiasm. They conjectured (their own word) that, if the numbers were arranged in 4 columns, the first column would have the 4s table; arranged in 5 columns, the first column would have the 5s table; arranged in 6, it would have the 6s table. They verified it already, looking at the tables. This group also discovered that:

- *In the diagonals, from left to right, the numbers grow a unit regarding the number of columns and in the diagonals from right to left, they decrease a unit.*

It was clear that they found more interesting to investigate what happened when the arrangement of the numbers was modified, than to discover the relations that existed between the numbers presented in the original figure.

I was somewhat concerned about what to do. One possibility was to leave them to continue even if in the final discussion they were not attuned with their colleagues. After all, doing investigations is not to go beyond what is predictable? Another possibility was to redirect them again towards the original task. In that case, the risk of killing their pleasure would not be too big? For a while, I let them follow their own way, taking some time to analyse their discoveries by myself (when preparing for the class, I had not thought about this type of exploration). This disturbed my attention towards the remaining groups of pupils.

The time for discussion was coming. I knew that the richness of the work of this group would not be understood by the others if they were to present their work immediately, since most pupils would be too much involved in the structure of the

task that they had been working. I thought, then, that the best way to value the work of these pupils was to give them some time to present it to the class in the next lesson. With some concern, I asked the group not to forget to also think a little about the initial figure.

In the next day, they made the presentation. The group chose Bruno to talk. Their classmates were very attentive. Some rivalry (barely disguised) hindered them to show great surprise with the discovery. However, the way they held the new task that Bruno proposed – *what happens when we modify the number of columns?* – made me think that they had understood that to investigate is to go beyond what is asked for; it is to feel free to explore things in our own way.

I finished the lesson with a mixed feeling of accomplishment and concern; accomplishment, because the pupils advanced in their understanding of investigations; concern, given the new challenge that I now face: how to handle a whole class where different groups of pupils move forward, in different directions, in their investigations?

LEARNING ABOUT INVESTIGATIONS

From the initial writing of this narrative we undertook its refining and analysis. In a first level of analysis, we tried to identify the main components of a narrative: summary, orientation, complication, evaluation, resolution, and *coda.* This narrative concerned a teacher who was excited about conducting an investigation class. She prepared the lesson carefully and started with a smooth presentation, to get pupils readily involved. Things got complicated when a group of pupils decided to ask new questions on their own, questions that the teacher had not thought about before. The pupils were very excited with their work but that meant a considerable departure from what the teacher had envisaged and felt comfortable doing.

The teacher evaluated the different possibilities. To leave pupils working on their questions would do justice to the idea of investigation. However, it would create problems in the class discussion that she regarded as a necessary stage of this activity. To redirect the pupils to the given task could hamper their interest and would be a disservice to the idea of investigation.

The teacher had to decide quickly. She opted for some compromise, albeit closer to the first option. She let the pupils work on their chosen path, but she also suggested that they not forget the original question. In this way, she hoped they could pursue their creative questions and strategies and, at the same time, follow the presentation made by their colleagues of their discoveries and, maybe, participate in the discussion.

The final reflection made by the teacher shows her sense of accomplishment with the activity. There was no reason for less, since the pupils worked hard, made many discoveries, showed understanding of what to do in investigations. They showed facility in using the investigative vocabulary using terms such as “conjecture”. Some pupils even went beyond what the teacher had planed and expected from them. This reflection also shows her concern with this kind of class, where management problems increase in an exponential way. Of course, this class

is harder to handle than a regular class where pupils work individually on exercises, and often requires the teacher to make difficult decisions regarding the direction of the activity.

In a second level of analysis we looked for other issues that appeared significant in the story and related them to other narratives. For example, this episode showed how investigations provide moments of intense involvement in mathematical thinking of pupils of different age levels and abilities. Other narratives produced in this project had many examples of such enthusiasm and of the richness of pupils' experiences – coming very close to the idea of a learning community. Investigations can bring life to the mathematics lesson, so often absent in other kind of activities.

Classes where pupils work on investigations show great learning potential. Pupils are clearly stimulated to think mathematically. Investigations also allow the establishment of connections among many topics, giving a coherent and integrated perspective of mathematics, completely different from the compartmentalised perspective that pupils tend to hold. They help to create a different – and truer – image of this science.

In this project, we strengthened our views regarding the importance of this type of mathematical work in the curriculum, at least in middle school mathematics. However, we are aware that the term "investigation" has diverse meanings, according to the authors, the contexts, and the traditions. The work that we carried out helped to clarify this concept. In the beginning, we were concerned about using investigations to foster learning of specific concepts. With the continuation of the work it became evident their value for the development of pupils' higher-order abilities and for a better understanding of their capacities and reasoning processes. Thus, we concluded that there are great advantages in open-ended tasks that give the pupils a chance to formulate their own questions. Regarded in this way, they are rather different from the routine work in "pattern spotting" and "table making" that in some countries is regarded as all of "investigating".

As we saw in this narrative, this sort of activity poses special problems to teachers. Preparing is very important. The task needs to be adjusted to the pupils' level and the teacher needs to have a good idea of the main directions they may follow in their explorations. Even when the teacher has planned well, surprises may still happen, and, as in this case, be difficult to handle. If the teacher is not prepared, he or she may be completely lost when students start moving in all kinds of directions.

Presenting the task to the class is another critical step. This teacher decided to give pupils some suggestions on aspects to consider. With other classes, such suggestions might be unnecessary or even counter-productive. Saying too much at this point may take the challenge off the pupils. Saying too little may hinder their understanding of the task and of what they were supposed to do. In addition, when pupils are working, the teacher needs to know how much support to give them. Overall, the role of the teacher must be rather unobtrusive, again supporting pupils without solving the problems for them. Things go well when the teacher challenges, poses questions and does not provide hints or direct responses. This requires a good assessment of students' needs and previous experience to decide what to do.

An investigation may end with a final presentation of conjectures, their justifications, implications, and connections. To do this, teachers need to know how to pose enlightening or stimulating questions that guide pupils without presenting clues. They need to demand contributions from all pupils and not just from those who usually have good ideas. Teachers also need to know how to evaluate pupils and provide them feedback regarding their accomplishments and shortcomings. For teachers, doing investigations in the classroom is more demanding and more laborious practice than many other kinds of activities.

LEARNING ABOUT COLLABORATION

As stated above, the success of collaborative work depends very much on the setting up of common goals and the negotiation of working processes that suit these goals and the different needs of all participants. Let us consider goals first. This collaborative action-research project intended to produce knowledge about the role of the teacher when pupils worked on investigations in the mathematics classroom and to produce and evaluate educational materials to support this kind of activity. There were three people involved, all acting as researchers, who wanted to understand the potential of students' investigations for mathematics learning and their implications for mathematics teaching and to communicate that to several audiences of teachers and mathematics educators.

The general aim of producing knowledge about conducting investigation classes was operationalised through the production of investigation tasks and supporting materials and the writing of papers with interesting experiences that were presented at professional meetings and published in educational journals. At some point, a more ambitious goal emerged—to produce a book with a collection of narratives concerning classes with pupils investigating mathematics and including a comprehensive discussion of the role of the teacher in that activity[3]. These operational goals provided many tasks for project members to carry out together in different moments and were a major factor of cohesion within the group.

The project strived to combine a strong ambition—making young pupils investigate mathematics, writing narratives about it, identifying key aspects of teachers' professional knowledge, and letting others know about it—with a realistic planning and use of resources. Doing this for an extended period led to a strong sense of doing something in common. A group identity emerged that did not stifle the participants' individuality but enriched it.

The different needs of the participants were also addressed in this project. The teacher and the two teacher educators had rather different backgrounds and personal concerns. The teacher was interested in getting support to prepare, conduct, and reflect about investigation classes. The teacher educators wanted to devise means of facilitating pre-service and in-service teachers' reflection about the professional knowledge necessary to conduct this kind of activity. These interests were accomplished with a lot of common work: preparing tasks, conducting and observing classes, sharing reflections, writing and refining narratives, and analysing them.

A COLLABORATIVE PROJECT: INVESTIGATIONS

The production and publication of materials related to investigation activities in the classroom and conducting in-service workshops involved all the participants in productive activity from the beginning of the project. The early discussion of theoretical articles was also useful in providing a common framework for conducting mathematical investigation classes. The commitment of the group to its goals was strengthened as we observed a favourable reaction from students to mathematical investigations; it was also reinforced as we saw that joint reflection, on the issues emerging from the narratives, helped us identify interesting things that happen in this kind of classroom activity.

The processes used in carrying out the project were adjusted according to the experience of the participants. There was a natural division of labour, taking into account the knowledge and interests of each member. Irene, as a classroom teacher, was in the best position to lead the experimenting of the tasks constructed by the project team. Hélia, as a teacher educator interested in the history of mathematics, made thorough searches for suitable starting points for investigations and studied what mathematicians wrote about them. João Pedro, as an experienced teacher educator, carried out the planning, reminded everybody about the next step, and did much of the editing of papers. Notwithstanding this division of labour, all project members were responsible for reading and commenting on the theoretical articles, proposing tasks, working towards their improvement, observing classes, writing stories and contributing to their critique and enhancement. All participated equally in sharing the ideas and products of the project with the educational community, writing materials, elaborating papers, doing presentations, and conducting in-service activities. There was an equal status for everyone in the group – all had to write texts, to criticise and to be criticised by the others, and to stand up for the project before several audiences. All contributed to the framing of the project questions, working procedures and products.

The production and analysis of narratives was a basic feature of the project methodology. Searching for the "complication" in each case took us to view the lesson under several perspectives which were important to understand the classroom activity and the teacher's role. For example, in the case reported in this paper the key issue was not clear at the beginning. It took a lot of discussions to finally isolate it and relate it to the remaining events as well as to develop the final reflection that closes the narrative. The production of the narratives was much more laborious than we expected for the main writer and for those who participated in the refinement process. A considerable effort was necessary. However, we found the process of narrative construction fruitful as it helped us to understand new aspects of the professional knowledge involved in investigation classes and in promoting the professional development of teachers who want to learn about it.

In all the activities carried out in the project there was a constant negotiation regarding aspects such as the objectives, nature, methodologies, timing, use of resources, and responsibilities. All participants were able to intervene in the group decisions. Perhaps the most salient feature of the activity of this project was the strong teamwork that progressively developed. The important decisions were always

discussed by all members and everyone recognises as his or hers the achievements and shortcomings of the project.

CONCLUSION

Besides their interest as a research methodology, narratives provide realistic cases that can be used in pre and in-service teacher education. In addition, they may be helpful for researchers, administrators, politicians, and parents—providing knowledge about what is going on in the classroom. The narratives produced in this project show that the teacher may be enthusiastic with pupils' creative mathematical work. They show a teacher involved in doing some mathematics and making thoughtful decisions regarding classroom work. They also demonstrate many examples of student involvement in mathematical activity and the value of teacher's reflecting about classroom activity.

Working collaboratively, a team composed by a teacher and two teacher educators was able to work for an extended period, developing some ambitious but realisable common goals. Each one brought to the group work their individual knowledge and experience, constructing together something that was quite different from the direct sum of the parts. Reflecting on the classes, looking at the complications, resolutions and evaluations provided new insights about doing investigations in the classroom. We conclude that this work involves quite demanding professional knowledge in selecting tasks that suit the pupils, in creating and sustaining a good classroom environment, conducting discussions, and taking the necessary decisions at critical moments of the work. We also note that this activity shows new aspects of the pupils' ability and these can be surprising for someone used to seeing pupils working mostly in routine tasks.

We suggest that investigations have an important role in pupils' mathematics education. To conduct such work requires a teacher interested in mathematics, capable of reflecting in his or her own practice, and willing to take risks. The teacher also needs resources, forums for exchange of experiences and sometimes direct support. Collaborative work with teacher educators and with other teachers is a natural way of involving teachers in such activity. It may provide an interesting inquiry context yielding new insights about the teaching activity.

NOTES

[1] This work is part of a larger project concerning pupils' mathematical investigations in the classroom called *Matemática para Todos: Investigações na Sala de Aula*, carried between 1995 and 1999 at Centro de Investigação em Educação da Faculdade de Ciências da Universidade de Lisboa and funded by Junta Nacional de Investigação Científica e Tecnológica and Instituto de Inovação Educacional.

[2] Part of this work was shared with other participants of *Matemática Para Todos* Project.

[3] The output can be seen in Ponte, J. P., Oliveira, H., Cunha, H., & Segurado, I. (1998).

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